COLLISIONLESS PHASE SPACE EQUILIBRIA IN PLASMAS

August 12, 2020

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PRINCETON PLASMA PHYSICS LABORATORY

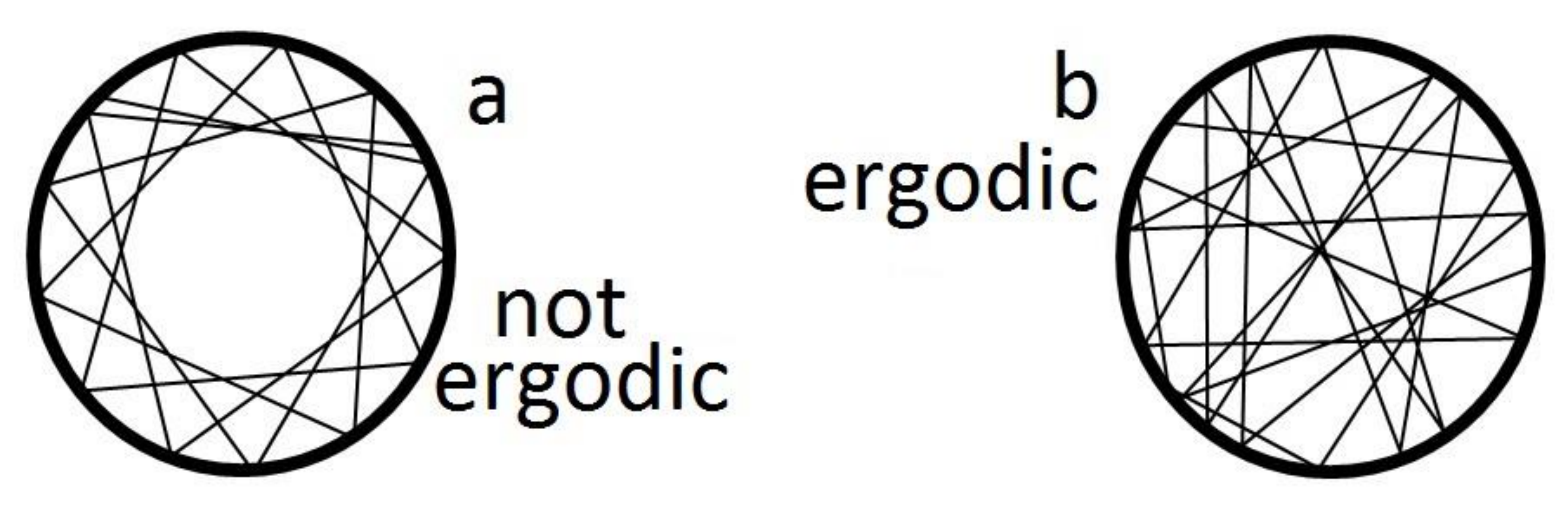
LEGES SINE MORIBUS VANAE

Goal

- Fusion Plasmas
- Solar Wind and most other astrophysical plasmas are weakly collisional (mean free path ~ 10^8 km)
- Measurements show *uniformity* and a Maxwellian-like velocity distribution
- When does the plasma equilibrate and how?
  - *Simulate this in 1x1v and 2x2v*

Statistical Mechanics

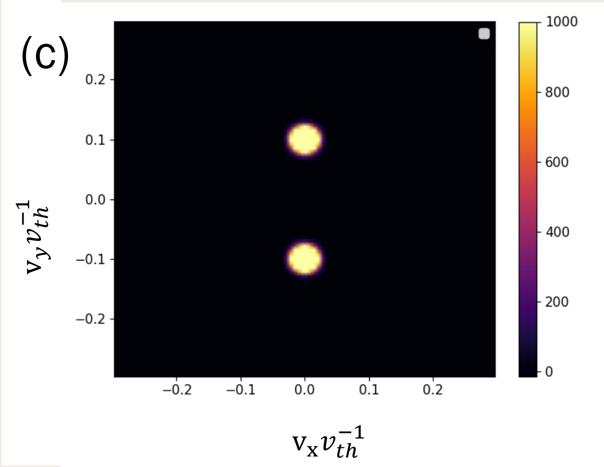
- Phase Space (configuration space of position and velocity)
- Ergodic Hypothesis
  - *Collisions*
- Lynden-Bell Equilibrium

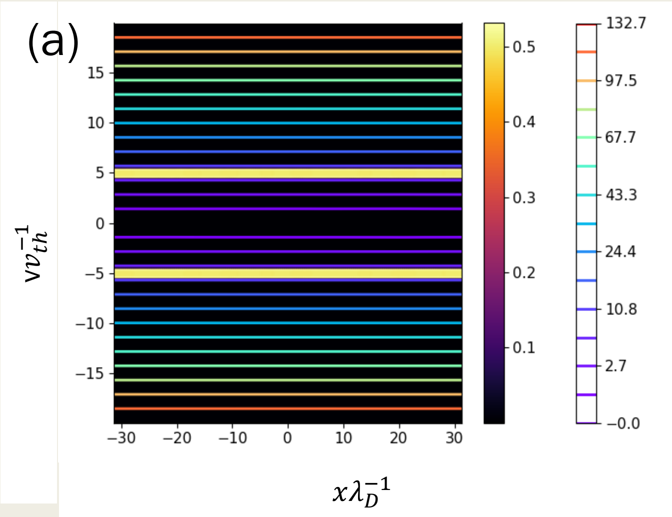


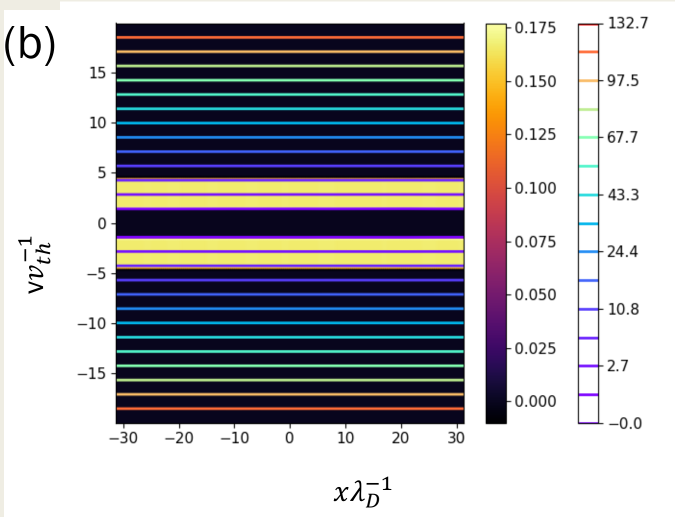
https://upload.wikimedia.org/wikipedia/commons/3/31/Ergodic_hypothesis_w_reflecting_rays.jpg

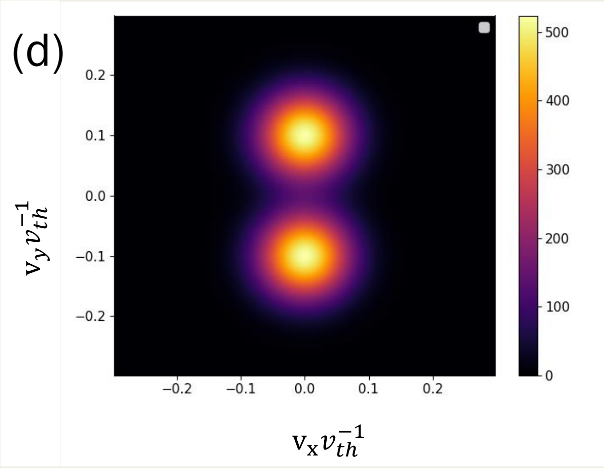
Instabilities

- In 1d there are two-stream instabilities (generate electrostatic waves)
- In 2d there are Weibel instabilities (generate electromagnetic waves)









Initialized States for (a) 1d cold waterbag, (b) 1d hot waterbag, (c) 2d cold Maxwellian, (d) 2d hot Maxwellian

Modelling Kinetic Plasmas with Gkeyll

- Discontinuous Galerkin
  - *Finite element and finite volume*
- Kinetic Modelling of collisionsless plasmas
  - *Vlasov-Maxwell on a Cartesian grid*
- Fitting simulation data with Stochastic Gradient Descent in PyTorch

$$\frac{\partial f_e}{\partial t} + \mathbf{v}_e \cdot \nabla f_e - e\left(\mathbf{E} + \frac{\mathbf{v}_e}{c} \times \mathbf{B}\right) \cdot \frac{\partial f_e}{\partial \mathbf{p}} = 0$$

$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla f_i + Z_i e\left(\mathbf{E} + \frac{\mathbf{v}_i}{c} \times \mathbf{B}\right) \cdot \frac{\partial f_i}{\partial \mathbf{p}} = 0$$

$$\nabla \times \mathbf{B} = \frac{4\pi \mathbf{j}}{c} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

$$\nabla \cdot \mathbf{B} = 0$$

Vlasov-Maxwell Equations

Collisions -> Exploring Phase Space -> Equilibrium
Entropy is maximized

But in the Vlasov-Maxwell framework, there are no collisions
and Entropy is a conserved quantity

Goal is to Use Simulations to Investigate the of physics of collisionless equilibration

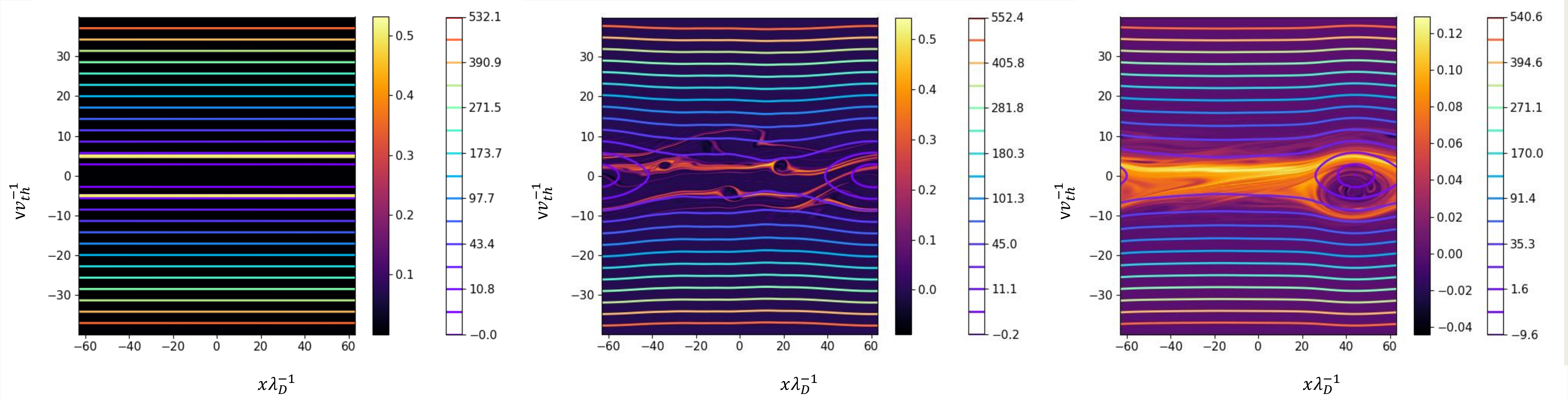
Vlasov-Maxwell Simulation of the Two-Stream Instability using Gkeyll

Results (1x1v Phase Space)

Initialized State

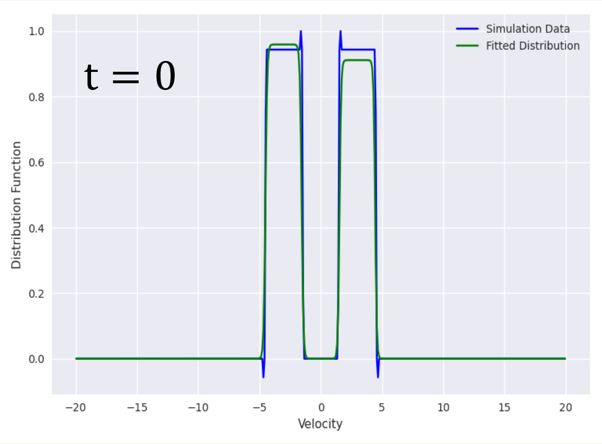
Instabilities Arise

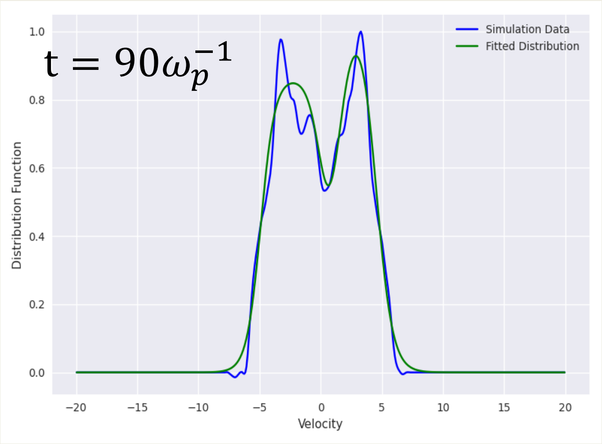
Equilibration

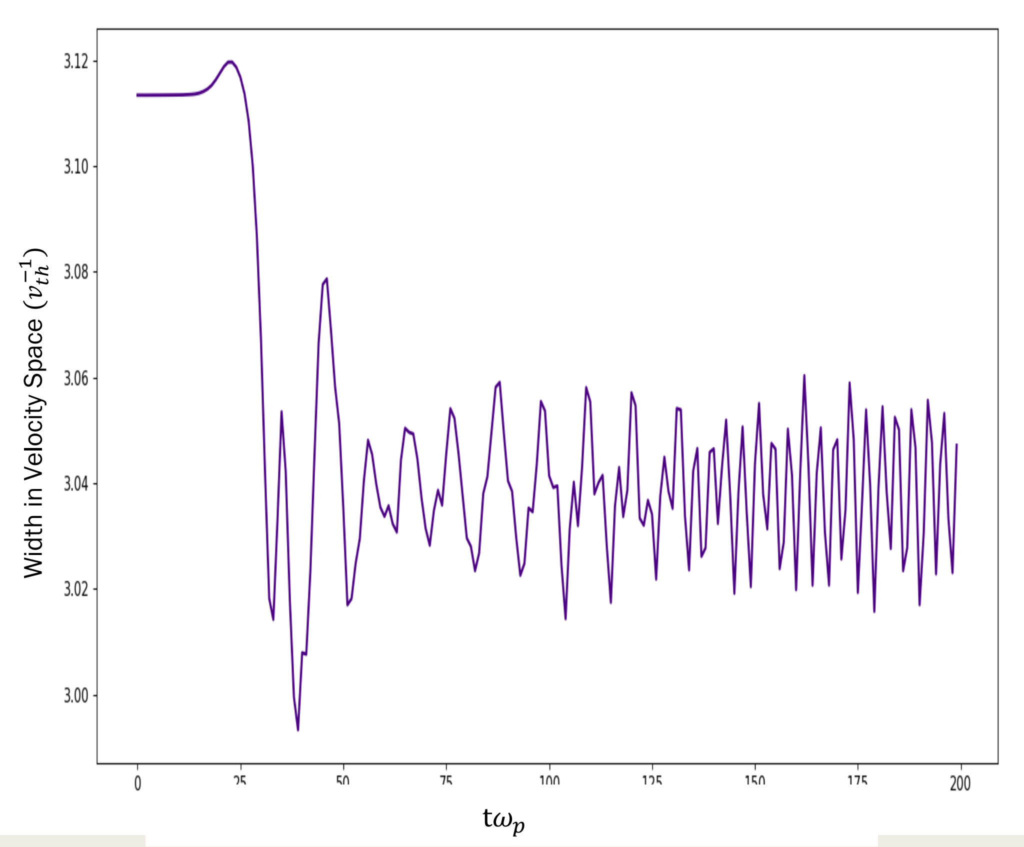


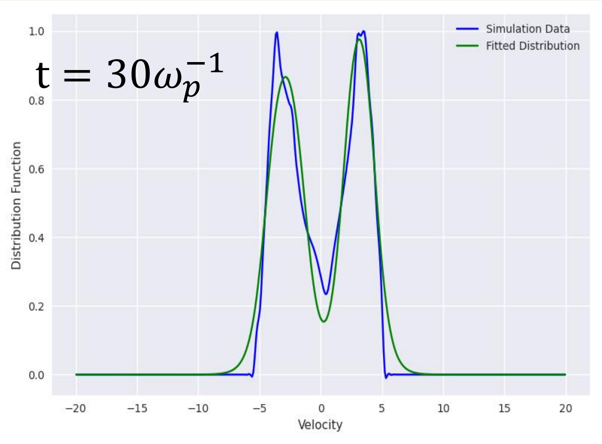
Probability Distribution Function plotted vs. position and velocity. Contours show the Hamiltonian isoenergic surfaces

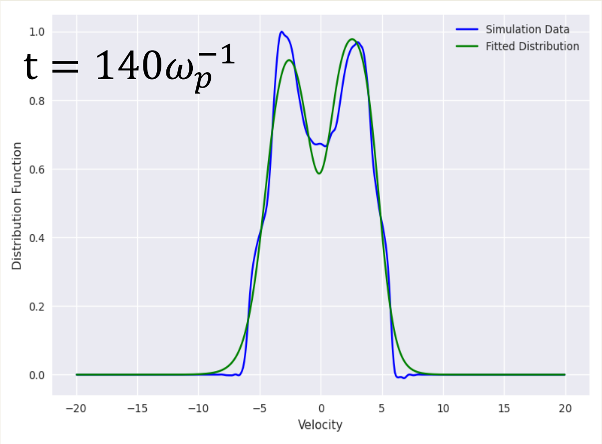
Fitting Distributions to the Data (1x1v)





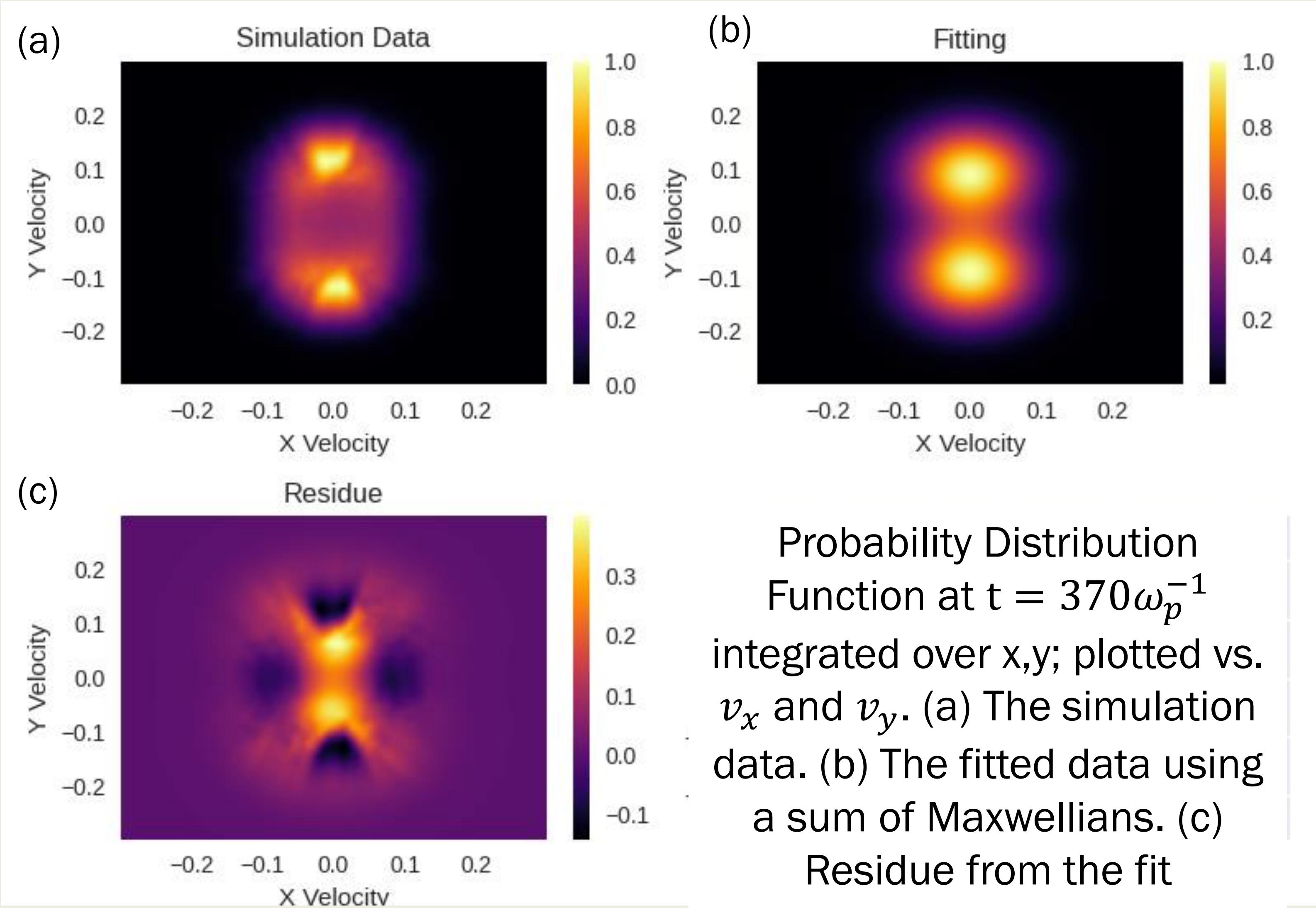






$$logistic\ distribution = \frac{h}{1 - \exp(-k(x - 2(\mu - \sigma))} + \frac{h}{1 - \exp(-k(x - 2(\mu + \sigma))}$$

Fitting Distributions to the Data (2x2v)



Probability Distribution Function at $t = 370\omega_p^{-1}$ integrated over x,y; plotted vs. v_x and v_y. (a) The simulation data. (b) The fitted data using a sum of Maxwellians. (c) Residue from the fit

Beam Remnants

- Fitted final few frames to a Maxwellian, and analyzed the residue from the fit, if it truly fits to a Maxwellian within an error, the residue should be a random distribution, but in fact, the residue is strongly correlated with the initial state, for the 1d simulation
- This contrasts with the 2d case where the residue from a Maxwellian fit is random

Conclusions and Relevance

- Implies dimensionality plays a significant role in the relaxation of collisionless plasmas
  - *Relaxation is more complete for 2d systems, may be due to the increased number of wave modes that are excited by 2d motion, electrostatic and electromagnetic wave modes*
- Significant for theoretical work as they typically examine simplified 1d systems, such systems may not capture relaxation behavior of higher dimensional systems

Acknowledgements

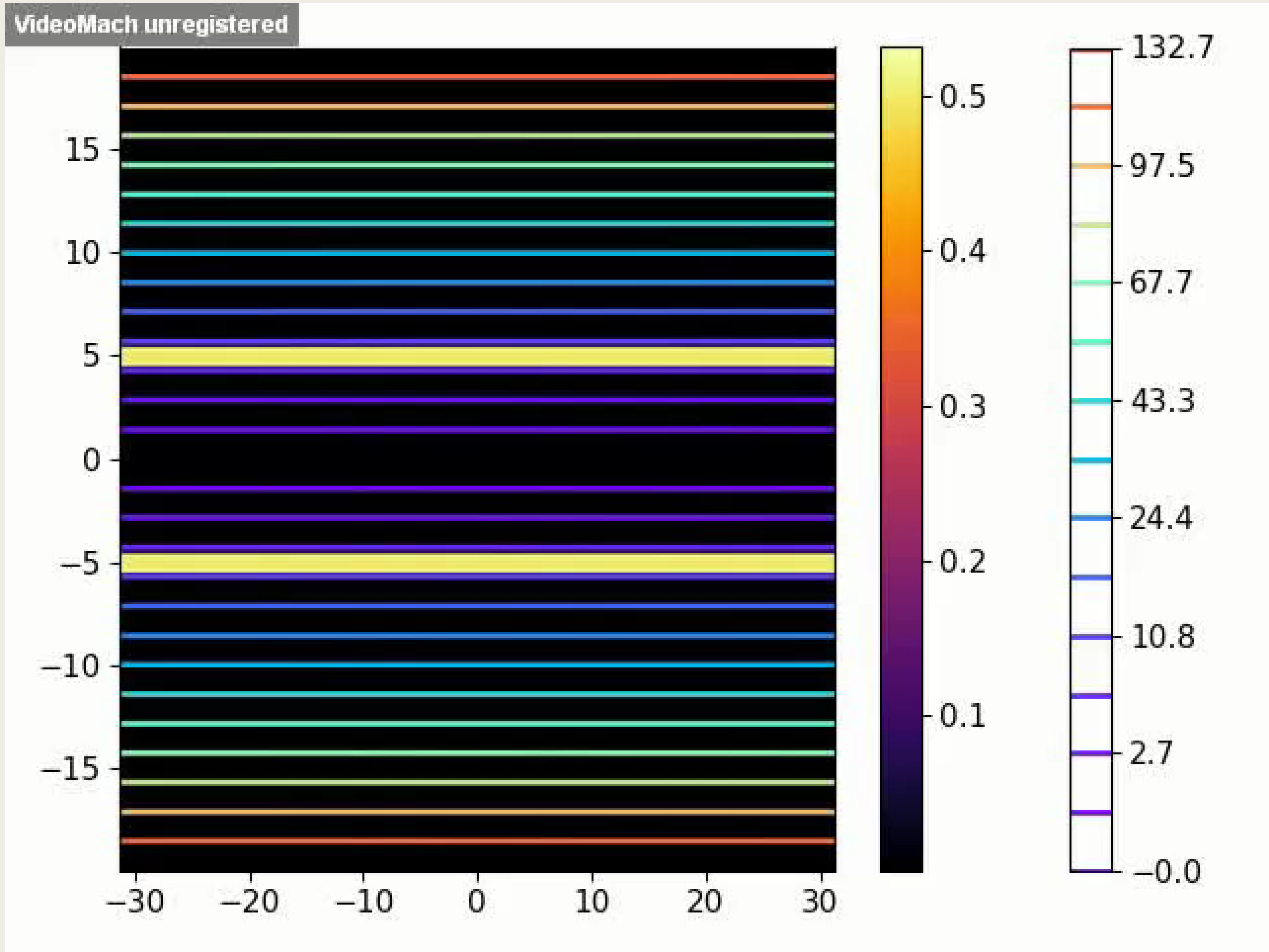
- This work was made possible by funding from the Department of Energy for the Summer Undergraduate Laboratory Internship (SULI) program. This work is supported by the US DOE Contract No. DE-AC02-09CH11466

THANK YOU!

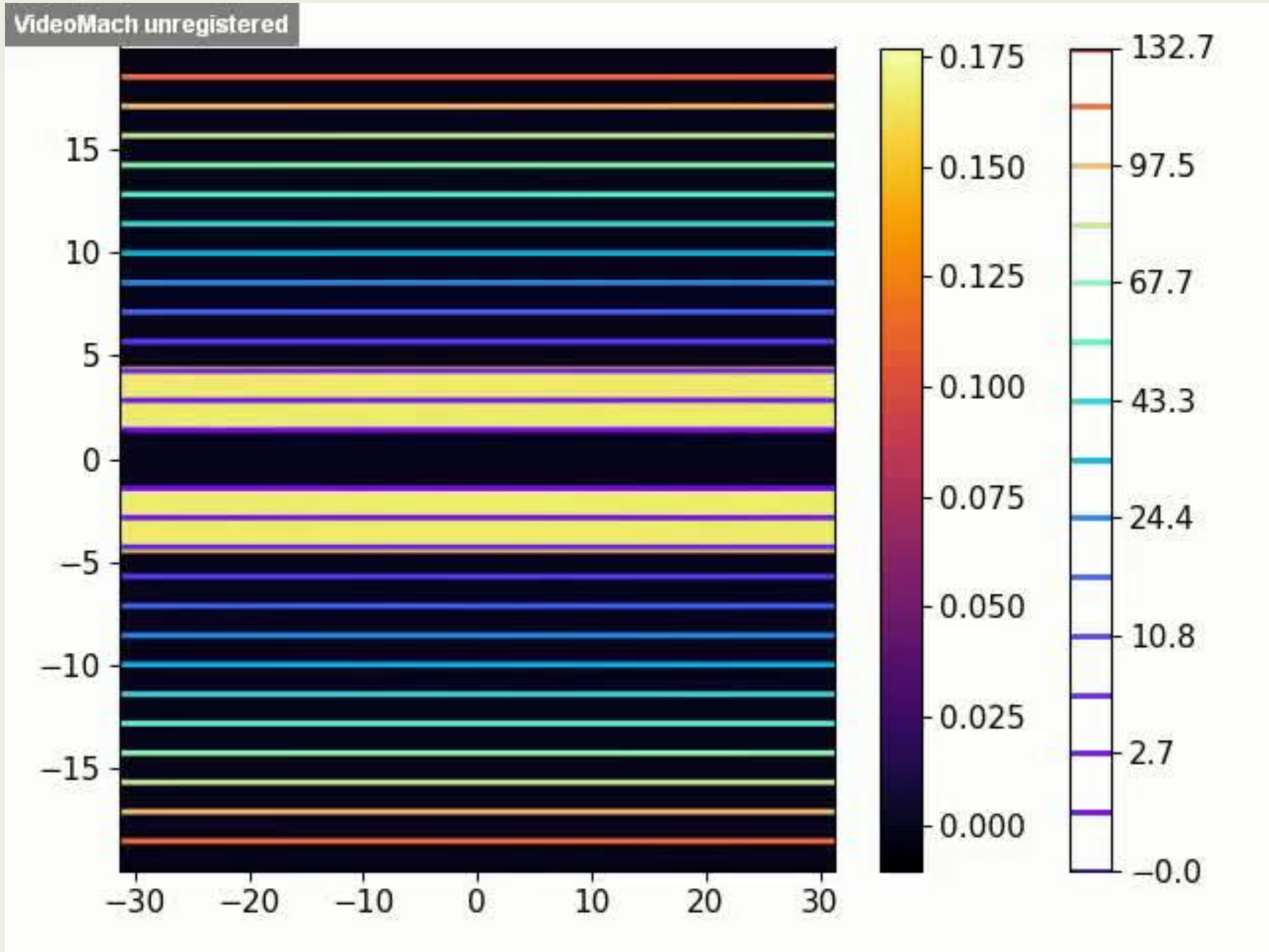
Questions?

SUPPLEMENTARY SLIDES

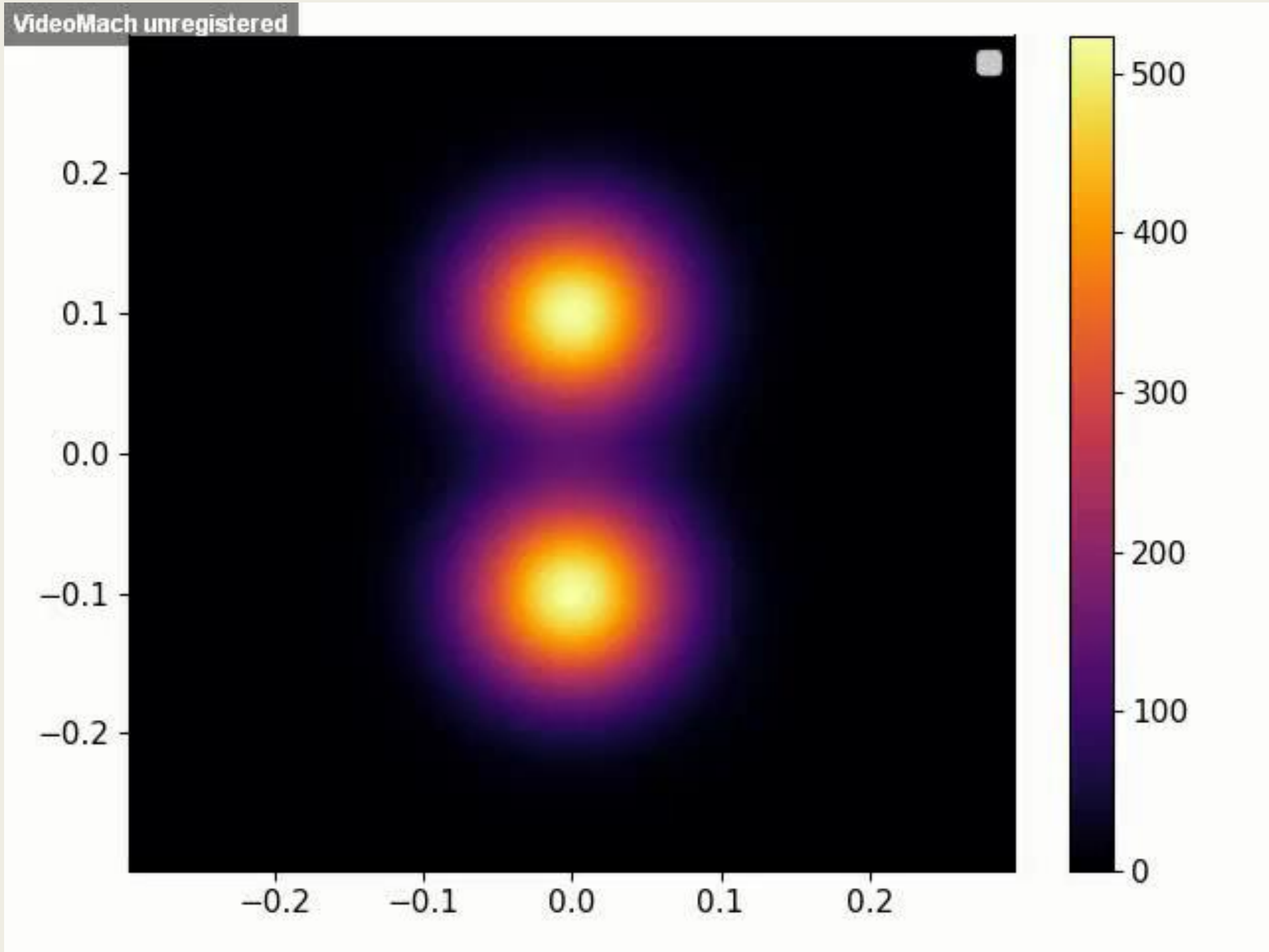
Cold Water Simulation Video with Isoenergic Surfaces (1x1v)



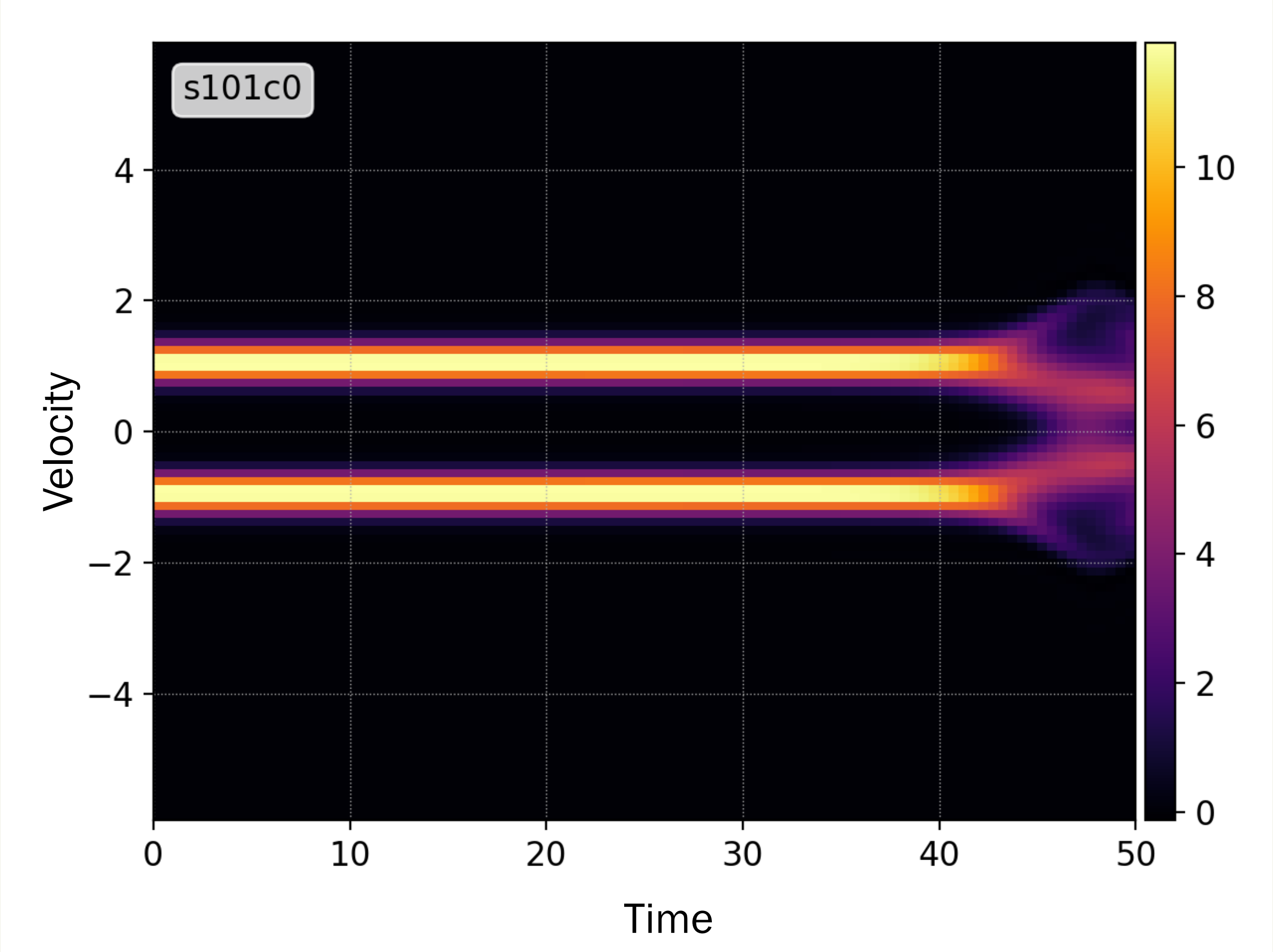
Hot Water Simulation Video with Isoenergic Surfaces (1x1v)



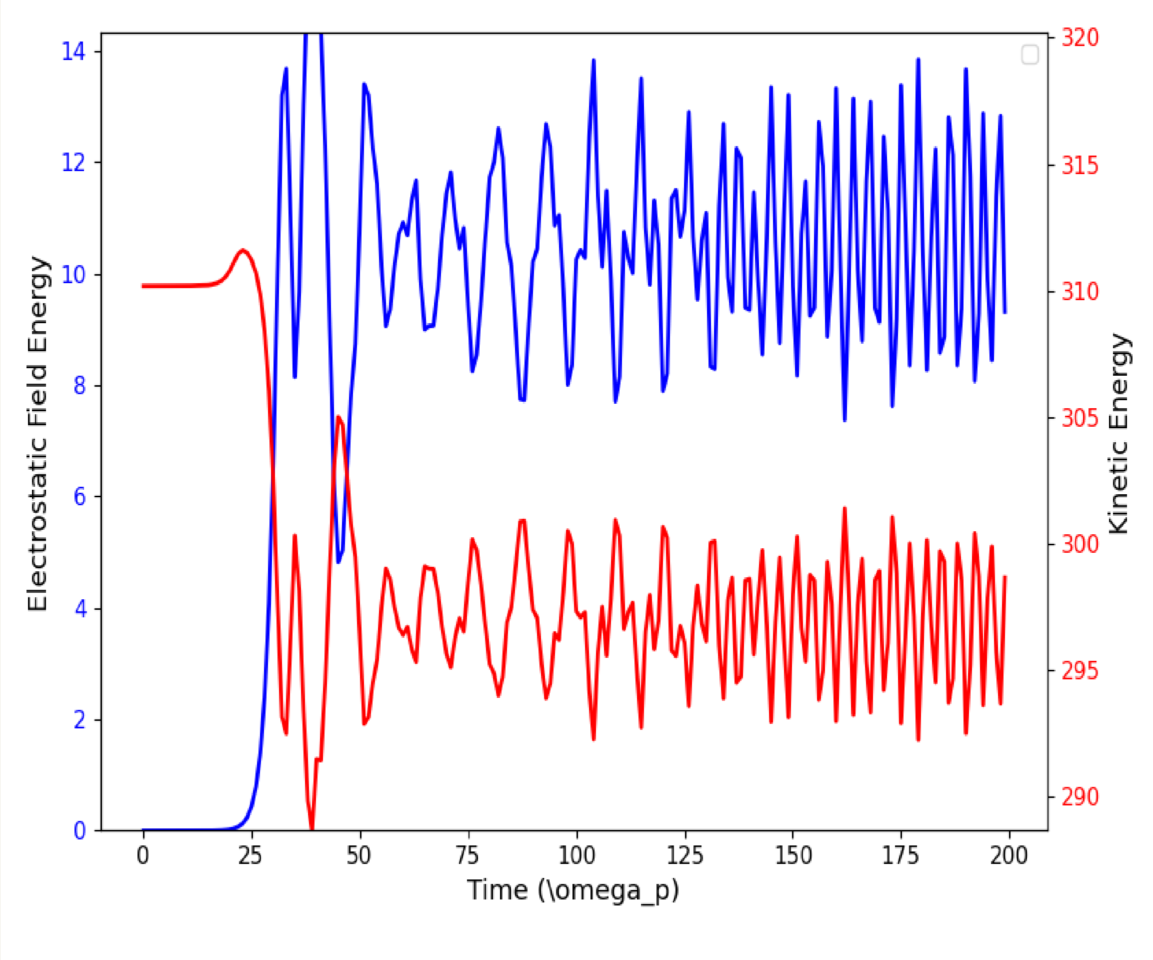
Hot Water Simulation Video (2x2v)

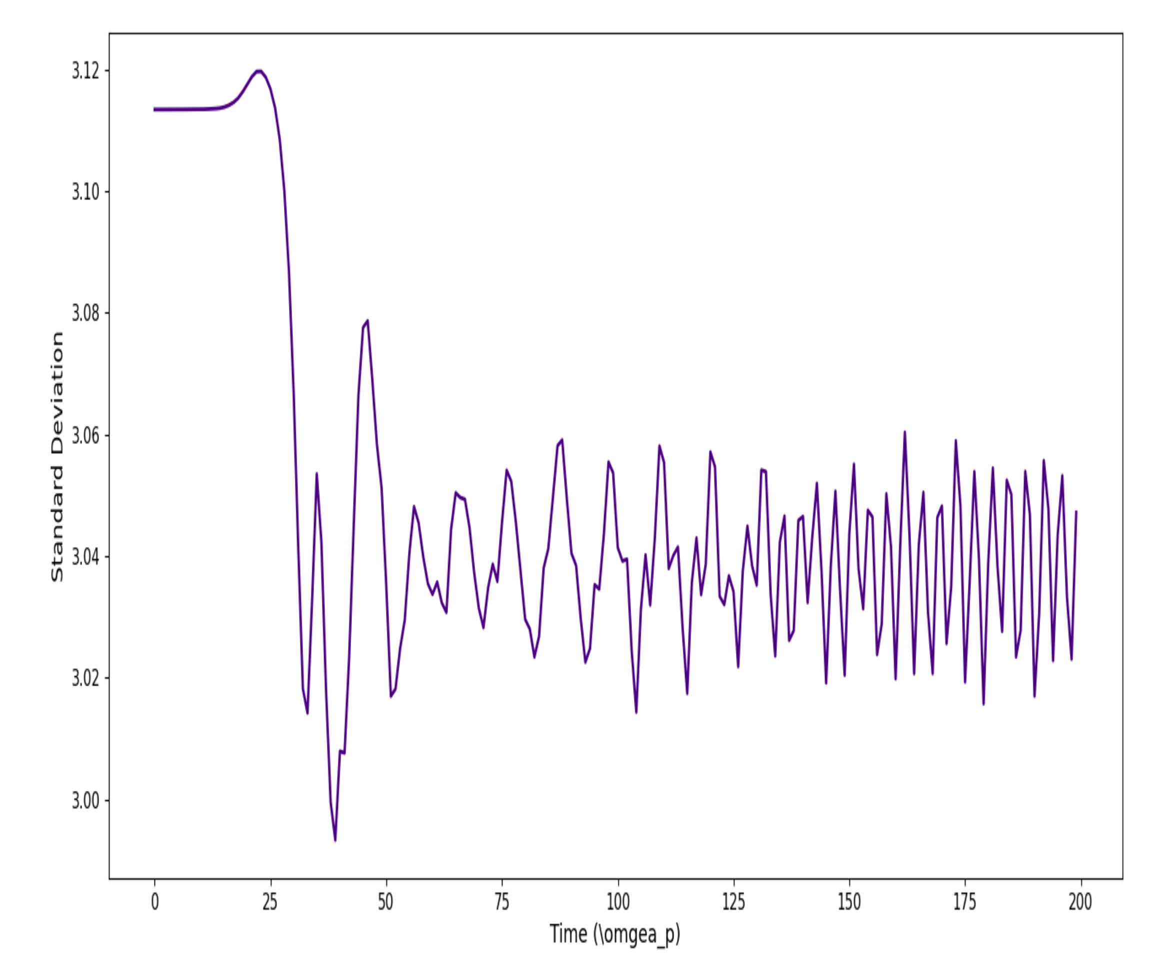


Computational Evidence Disagrees, There is Phase Space Mixing

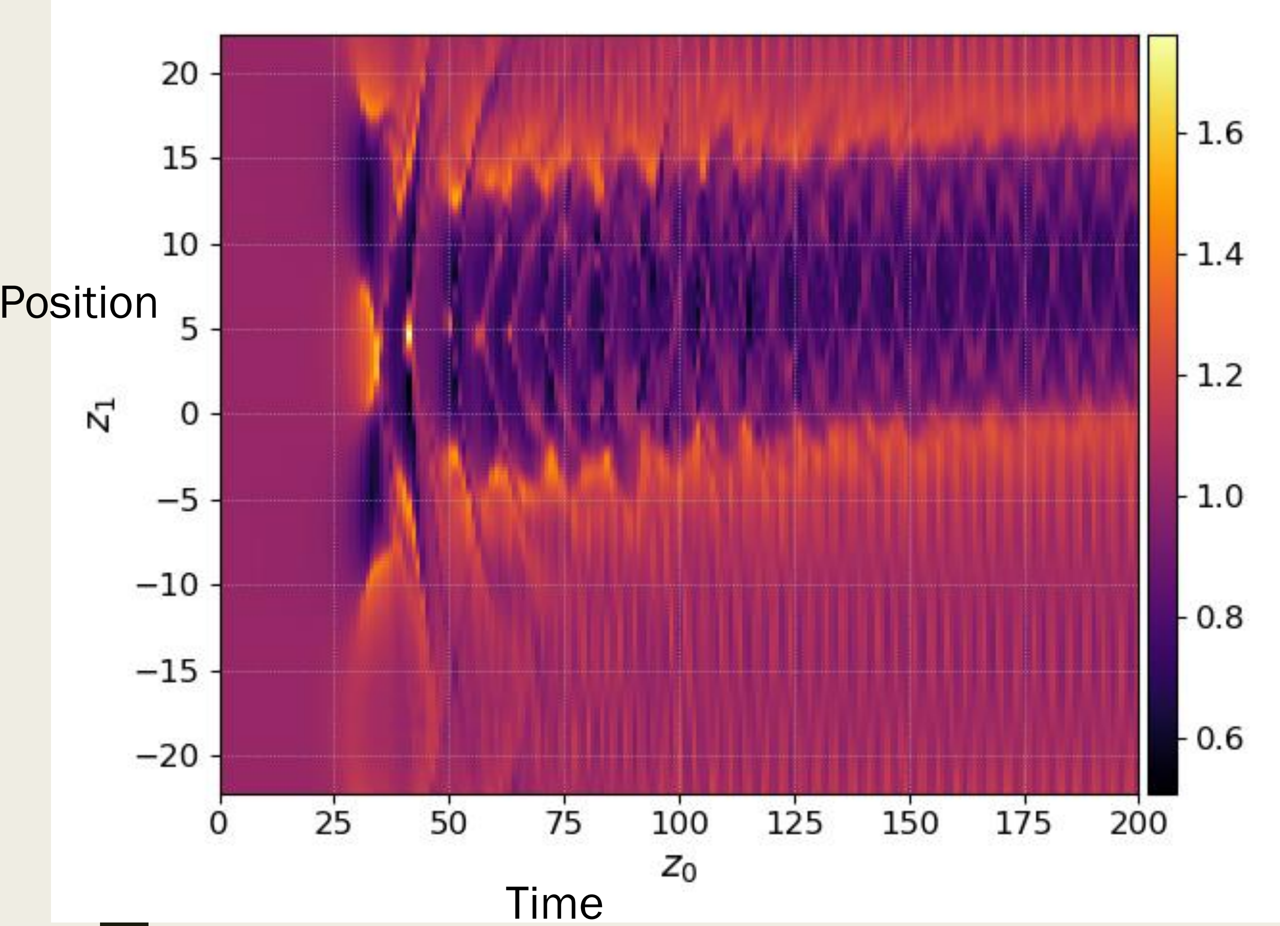


Saturation and Equilibration (1x1v)

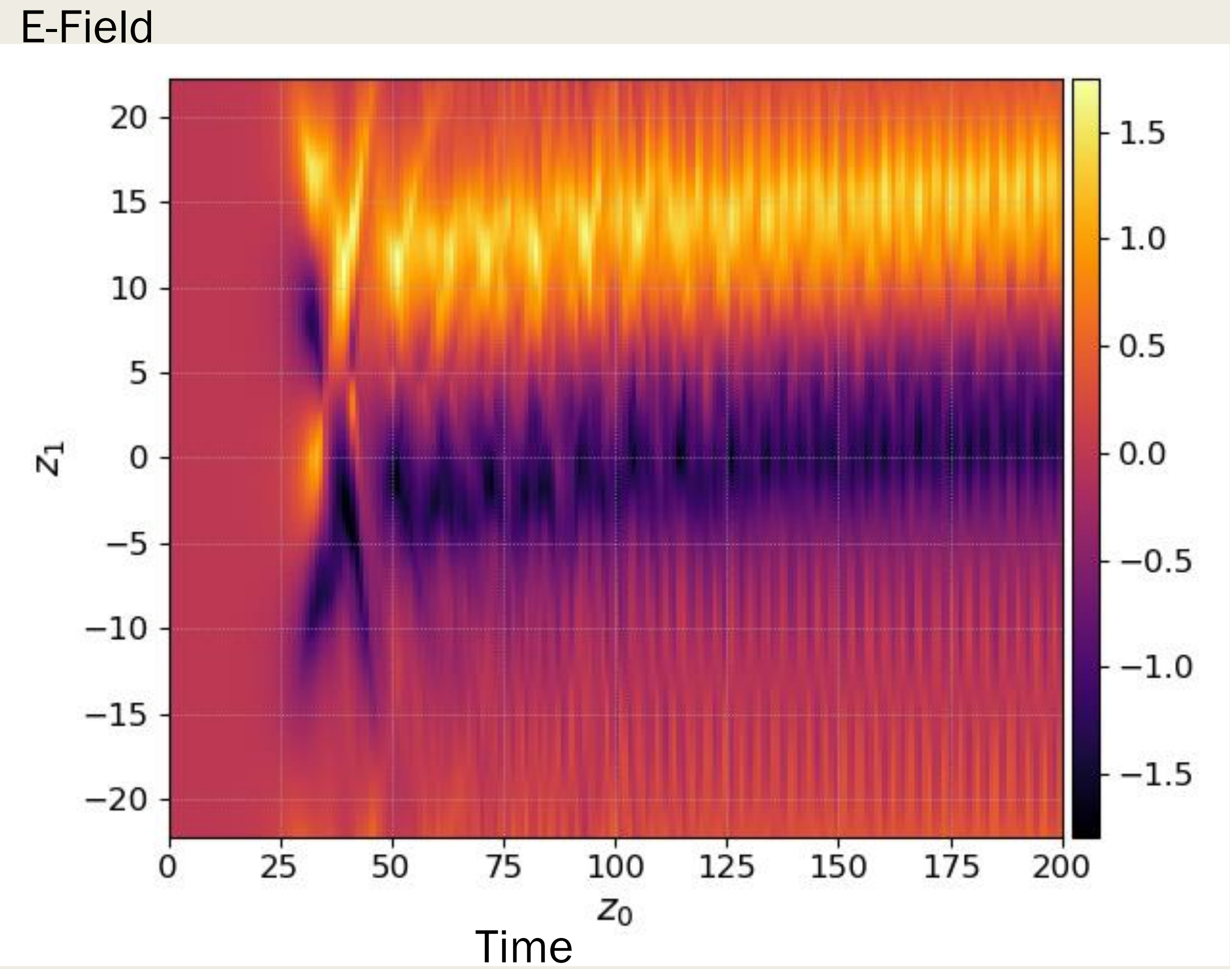




Two-Stream Instability



BGK Mode

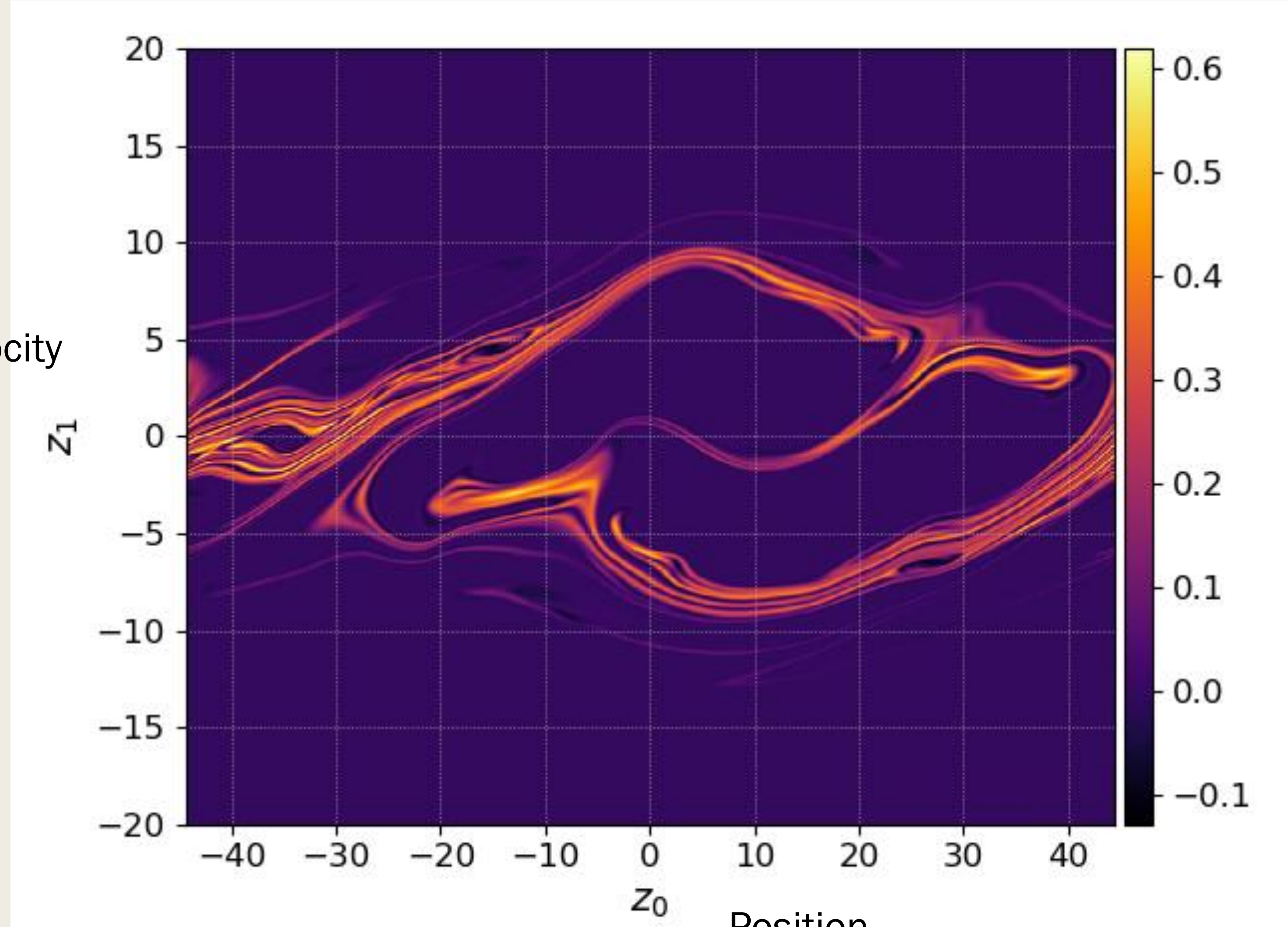


About Me

- Rising Senior at the University of Pennsylvania majoring and submatriculating in Physics and minoring in Mathematics and German
- Working with Jason TenBarge at PPPL

Results (1x1v)

Velocity



Position